Application of AIRS v5.0 Averaging Kernels

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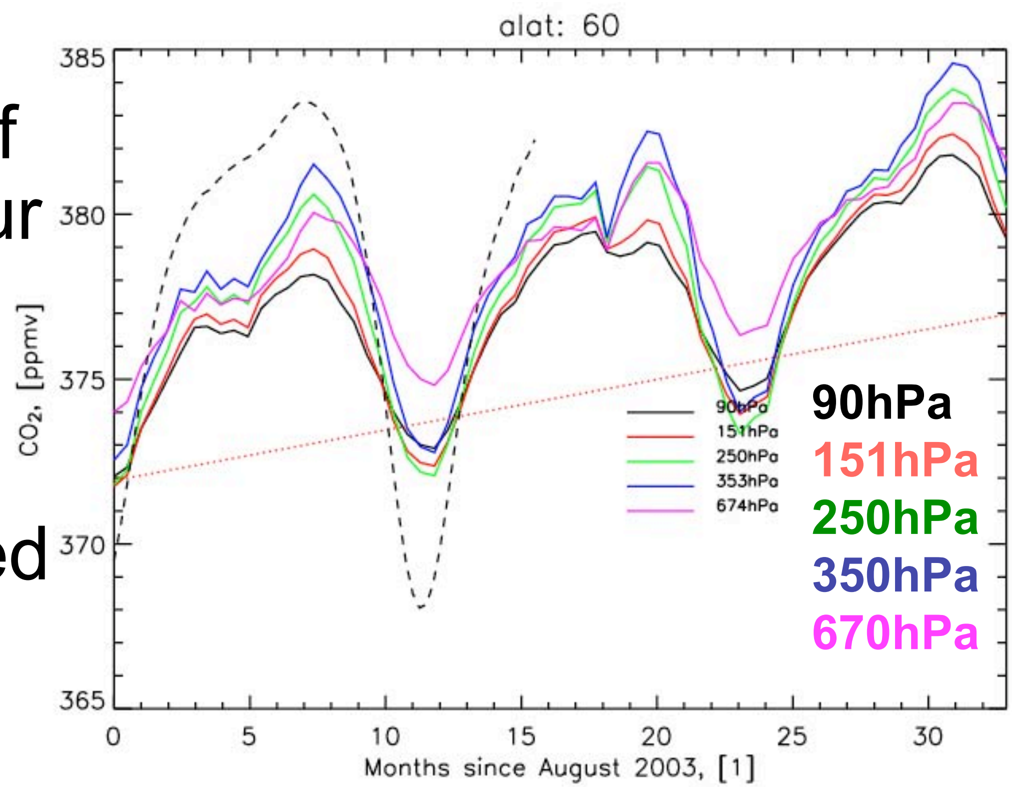
Outline

- Provide an overview of v5.0 averaging kernels/smoothing operators
  - What are they?
  - How do we apply them and what are the caveats?
- Discuss diagnostic capability of averaging kernels
  - Calculation of retrieval resolution
    - Averaging kernel resolution
    - FWHM error covariance matrices
  - Calculation of statistics using averaging kernels
- Summary and Future Directions

Biweekly CO_2 from AIRS 3°x3° grids and NOAA ESRL/GMD MBL CO_2

- Damping of the amplitude seasonal cycle as a function of pressure is due to our vertical sensitivity of the product.
- This information needs to be conveyed to modelers and the general user community.



What are averaging kernels?

- Averaging kernels are a linear representation of the vertical weighting of retrievals.
  - Related to the amount of information determined from the radiances and how much is due to the first guess [Rodgers, 1976].
    - To some degree avoids aliasing comparisons of in situ measurements vs. retrievals due to incorrect first guesses.
    - Enables assessment of where vertically we have information.
  - Related to the vertical resolution of retrievals [Backus and Gilbert, 1969; Conrath, 1972; Rodgers, 1976; Purser and Huang, 1993]
  - Required by modelers to properly use AIRS trace gas products.
  - Enables assessment of retrieval skill on a case by case basis.
- In the IDEAL case (no damping): $A = I$: the identity matrix

Averaging Kernels Limitations

- Our averaging kernels are a conservative estimate of the vertical correlation of products because the startup regression solution ($T/H_2O/O_3$) has it's own averaging kernel.
  - This becomes important only when our products are overdamped.
  - We (NOAA) have the ability to calculate this averaging kernel for case studies if necessary.

- Iteration (esp. background term)/stepwise retrieval complicate interpretation
  - There is a cross-talk between averaging kernels that is not addressed properly.
    - The temperature retrieval believes a fraction of the radiances so that the averaging kernel for products does not exactly relate to the amount of the radiances believed.
    - Separation of signals using propagated noise covariance terms as well as intelligent selection of channels minimizes this effect.
  - Non-linearity (I won't go into this too much here) is not properly handled by the linear averaging kernel analysis.

Averaging Kernels Limitations

- Vertical weighting is strictly defined on the retrieval grid, not the RTA grid.
  - Any estimate of resolution based on the internal averaging kernels is limited by the resolution of our retrieval functions.
  - Transformations between retrieval functions and AIRS layers exist; however they assume that we can "upsample" derivatives without loss of accuracy.
    - Not a big problem if we have sampled the atmosphere adequately with respect to channel temperature and gaseous kernel functions.

A Note on Trapezoidal Functions

- Trapezoidal functions (denoted, $\mathbf{F}_{L,j}$) are used to interpolate retrieval delta's onto the RTA grid:

Fine level/layer retrieved quantities interpolated onto RTA grid.

$$\Delta \mathbf{x}_L = \sum_j \mathbf{F}_{L,j} \Delta \mathbf{A}_j$$

Coarse layer retrieved quantities

- These functions serve two purposes:
  - Define a reduced measurement space on which finite difference derivatives are calculated.
  - Ensure a smooth product (interpolation).
- Transformation between RTA grid and coarse layers is provided by a least squares estimate:

$$\Delta \mathbf{A}_j = \sum_{L'} \mathbf{F}^{+}_{j,L'} \Delta \mathbf{x}_{L'} = [\mathbf{F}^{T}_{j,L} \mathbf{F}_{L,j'}]^{-1} \mathbf{F}^{T}_{j',L'} (\mathbf{x}_{L'} - \mathbf{x}_{0,L'})$$

Least squares estimate requires halfbot and halftop forced to .false.

Linear vs. Log derivatives

[Rodgers and Connor 2003] form of the equation assumes linearity in changes in state. For temperature this is true and we have:

First Guess Truth

$$\mathbf{x}' = \mathbf{x}_0 + \ddot{\mathbf{O}} \cdot \left(\mathbf{x} - \mathbf{x}_0\right)$$

Convolved truth Averaging Kernel

For minor constituents (H_2O, O_3, CO, CH_4, etc.) the averaging kernels act in logarithmic or %changes in state:

$$\log(\mathbf{x}') = \log(\mathbf{x}_0) + \ddot{\mathbf{O}} \cdot \log(\mathbf{x} / \mathbf{x}_0)$$

For small perturbations/low information content we can write in terms of % changes relative to the first guess:

Unit vector

$$\mathbf{x}' = \mathbf{x}_0 \left[\mathbf{1} + \ddot{\mathbf{O}} \cdot \left(\frac{\mathbf{x} - \mathbf{x}_0}{\mathbf{x}_0} \right) \right]$$

Retrieval Functions and Convolution Recipe

The retrieval calculates coarse layer derivatives and assigns retrieved changes to fine layers using slb2fin (trapezoids denoted $\mathbf{F}_{L,j}$).

We can handle the trapezoidal retrieval functions in much the same way that the retrieval handles them by:

1. Calculating coarse layer delta states. *e.g.,*

$$\Delta\mathbf{A}_j = \sum_{L'} \mathbf{F}^{+}_{j,L'} \Delta\mathbf{x}_{L'} = [\mathbf{F}^{T}_{j,L}\mathbf{F}_{L,j'}]^{-1}\mathbf{F}^{T}_{j',L'}(\mathbf{x}_{L'} - \mathbf{x}_{0,L'})$$

2. Apply averaging kernel to coarse layer deltas and use the functions to interpolate to the RTA grid.

Minor gases:
Let: x = log(x)

$$\Delta\tilde{\mathbf{x}}_L = \tilde{\mathbf{x}}_L - \tilde{\mathbf{x}}_{0,L} = \sum_j \mathbf{F}_{L,j} \cdot \sum_{j'} [\Phi_{j,j'} \cdot \Delta\mathbf{A}_{j'}]$$

3. `Use convolution equation on interpolated convolved delta state:

$$\mathbf{x}' = \mathbf{x}_0 + \Delta\tilde{\mathbf{x}}$$

Retrieval Smoothing Terms

- Retrieval smoothing is composed two terms:
  - Regularization (*e.g.* a noise threshold value termed B_{max}).
  - Trapezoidal interpolation rule.

Trapezoidal Smoothing

$$\Delta\hat{\mathbf{x}}_L = \mathbf{F}_{L,j} \cdot \underbrace{\ddot{\mathbf{O}}_{j,j'}}_{\text{Averaging Kernel Smoothing}} \cdot [\mathbf{F}^T_{j',L}\mathbf{F}_{L,j}]^{-1}\mathbf{F}^T_{j,L'}\Delta\mathbf{x}_{L'}$$

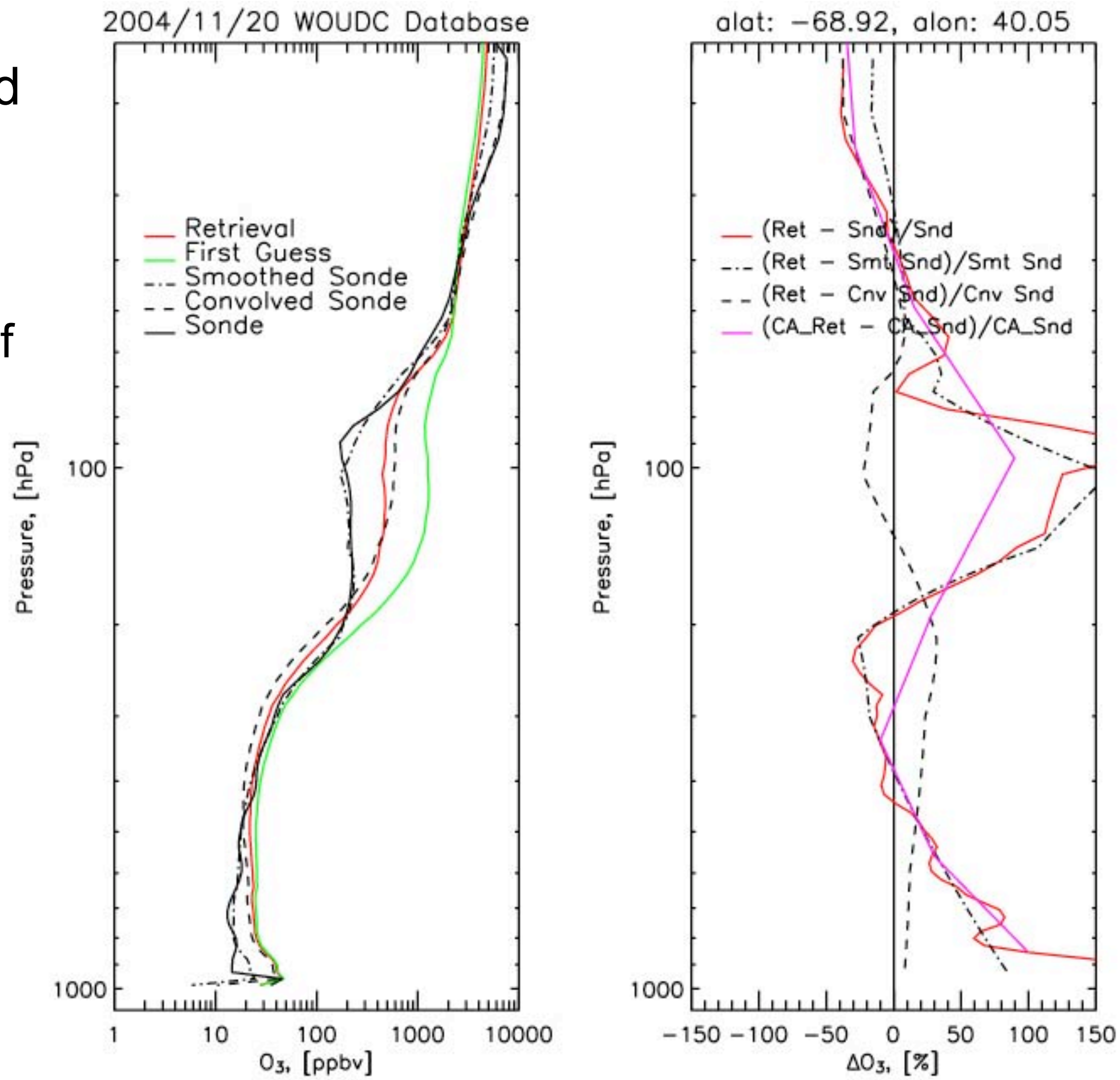
Averaging Kernel Smoothing

- Regression can impart high resolution structure, this structure is removed from the comparison by the trapezoidal smoothing terms if it is finer than the width trapezoids.
- The following slide illustrates each component.

An example of retrieval smoothing and convolution (O_3 hole S. Pole)

- Smoothed sonde calculated assuming averaging kernel = identity matrix
  - Ideal case -- what we would do in the absence of damping.
- Convolved sonde using case dependent averaging kernel.

Retrieval and Convolved Sonde Compare very well.



Trapezoidal Null Space

- Projecting the truth-fg onto the trapezoids and interpolating onto the RTA grid.

$$\Delta\tilde{\mathbf{x}}_L = \tilde{\mathbf{x}}_L - \tilde{\mathbf{x}}_{0,L} = \mathbf{F}_{L,j} \cdot \mathbf{F}^{+}_{j',L'} \cdot \Delta\mathbf{x}_{L'}$$

$$\Delta\tilde{\mathbf{x}}_L = \mathbf{F}_{L,j'} \cdot \mathbf{F}^{+}_{j',L'} \cdot \Delta\mathbf{x}_{L'}$$

$$\Delta\tilde{\mathbf{x}}_L = \mathbf{F}_{L,j'} \cdot \mathbf{F}^{+}_{j',L'} \cdot \mathbf{F}_{L',j} \cdot \Delta\mathbf{A}_j$$

$$\Delta\tilde{\mathbf{x}}_L = \Delta\mathbf{x}_L$$

Components of the trapezoidal smoothing error are zero if the difference between the first guess and “truth” can be written as a superposition of trapezoidal perturbations!

- Standard deviation between smoothed truth and truth (note this is dependent on the trapezoid spacing, variability in the truth and variability in the first guess).

	F^+	Slab avg.
T(p)	0.25K-0.5K	0.5K-1.0K
H_2O(p)	5%-10%	10%-20%
O_3(p)	5%-10%	10%-20%

Resolution estimates from error covariance matrices and averaging kernels

- Vertical resolution of any retrieval is related to the width of the kernel functions and hence averaging kernels.
  - Backus-Gilbert, 1969
  - Conrath, 1972
- We can also define the vertical resolution in terms of the error correlation between atmospheric layers.

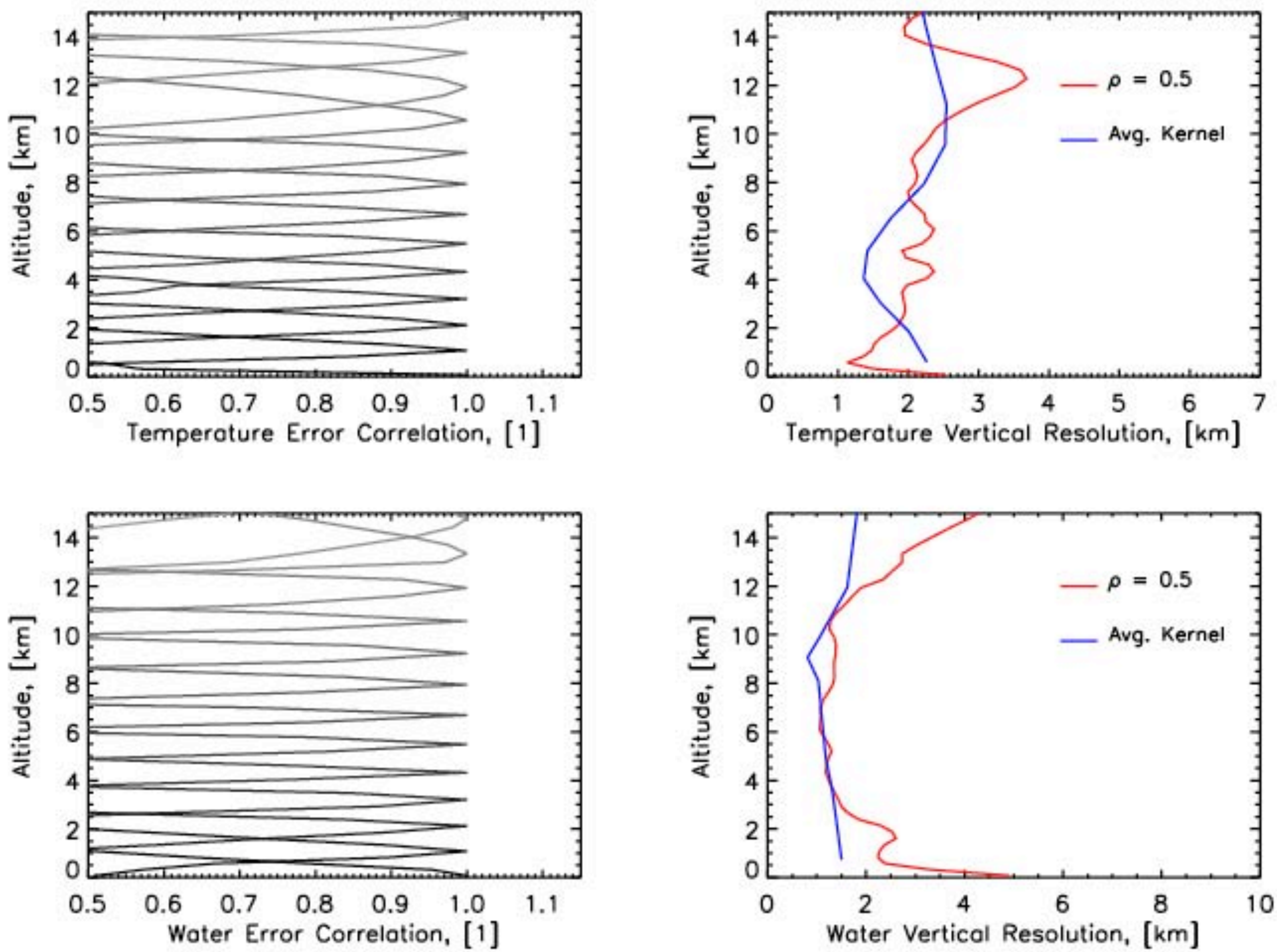
Truth value at RTA grid index, *i*

$$\tilde{n}_{i,j} = \frac{\operatorname{cov}(\Delta \mathbf{x}_i, \Delta \mathbf{x}_j)}{\sigma_i \sigma_j}; \quad \Delta \mathbf{x}_i = \hat{\mathbf{x}}_i - \mathbf{x}_i$$

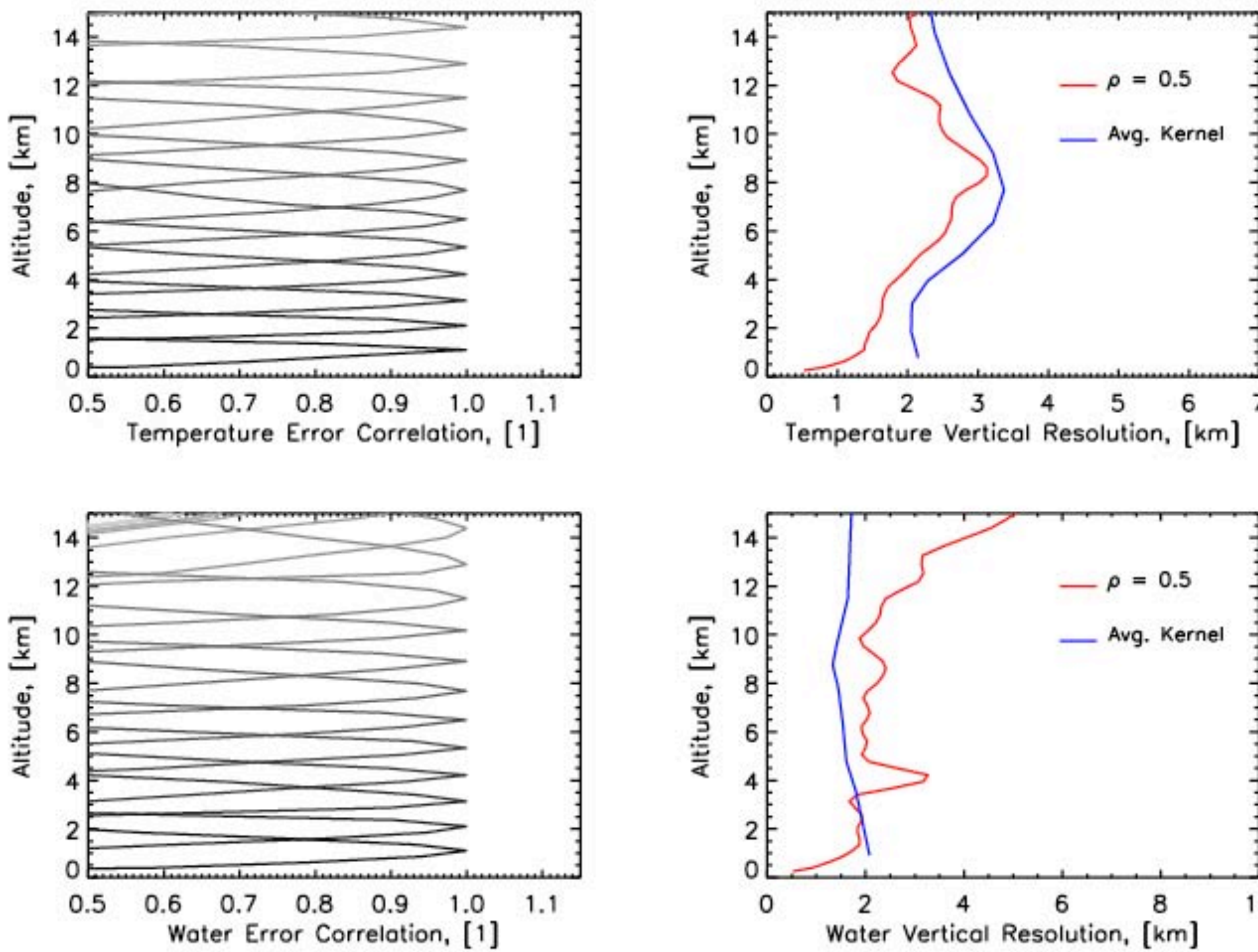
Error correlation matrix

Retrieved value at RTA grid index, *i*

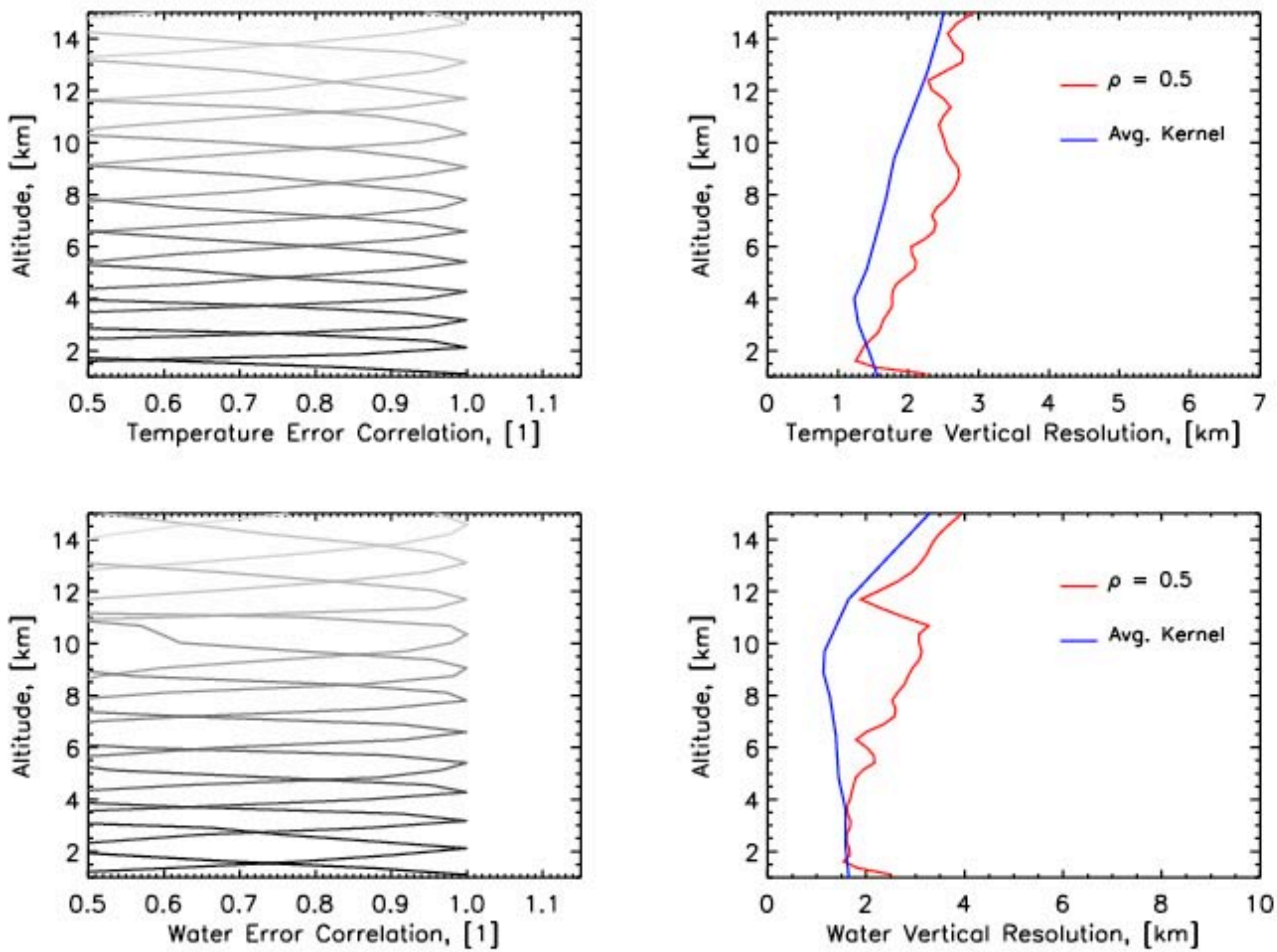
Vertical correlation and resolution at ARM-TWP



Vertical correlation and resolution at ARM-SGP

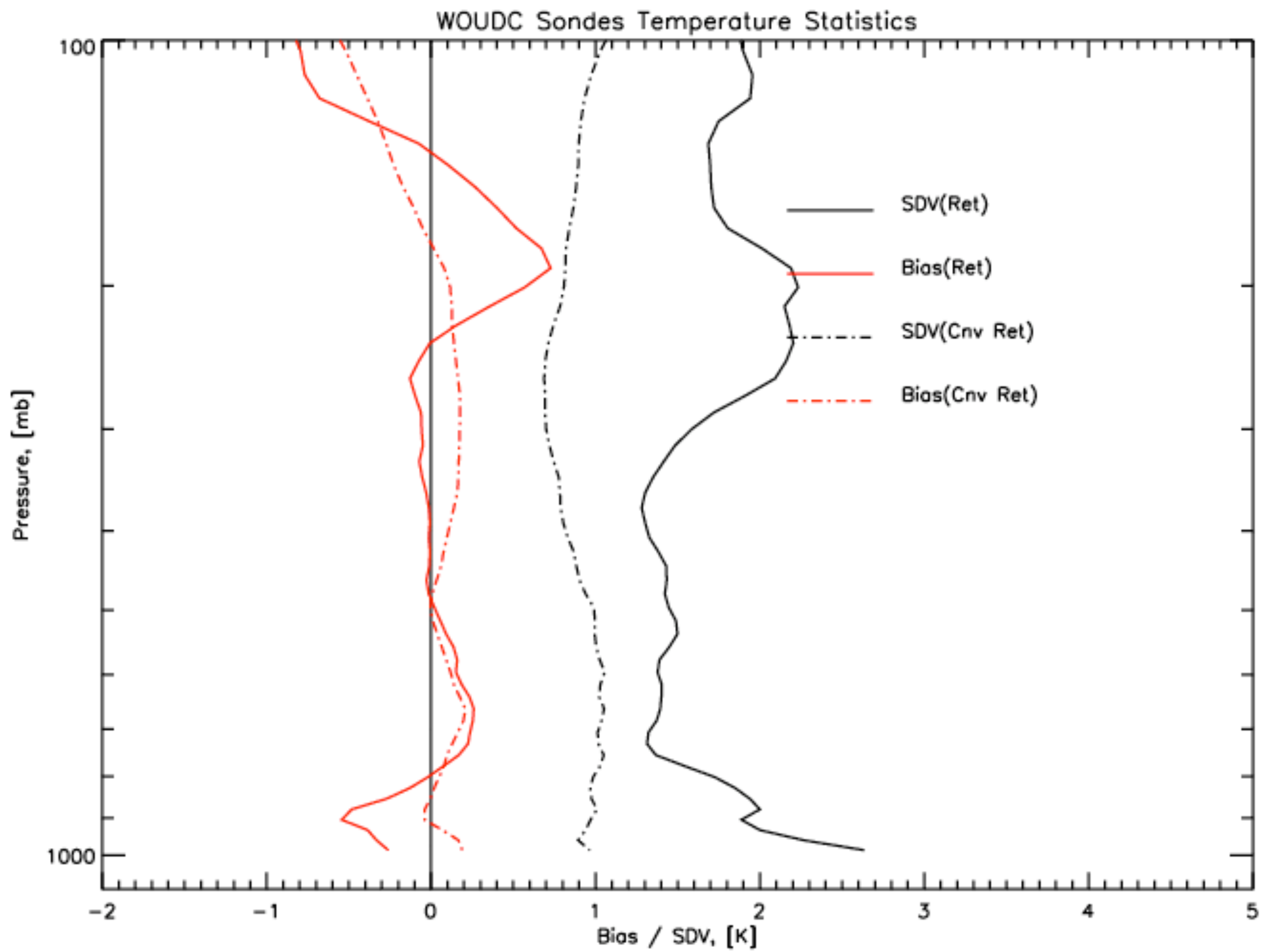


Vertical correlation and resolution in NOAA sondes

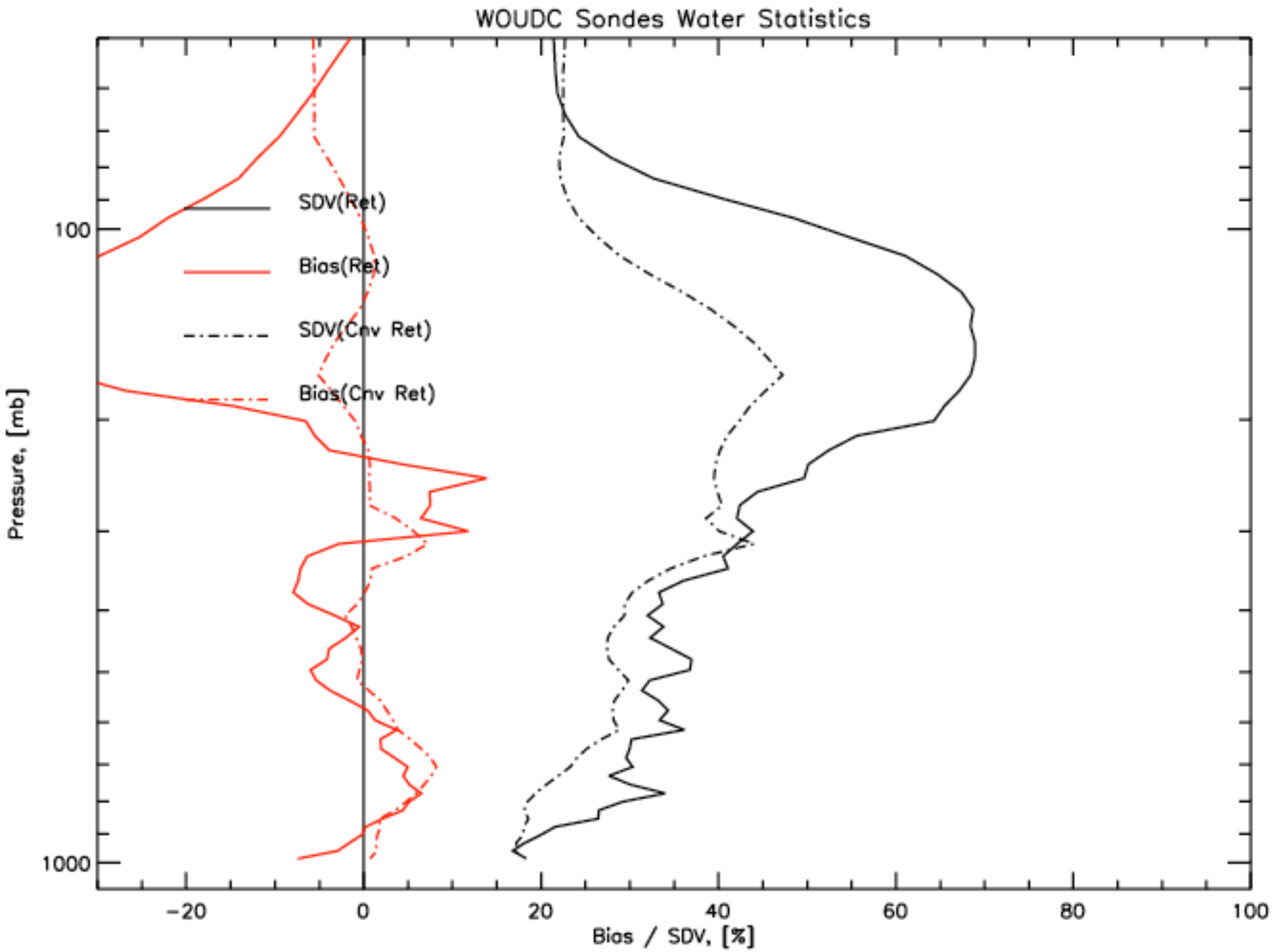


Examples of Statistics using Averaging Kernels

- The information content of AIRS spectra is highly scene dependent (*e.g*. clear *vs.* cloudy, tropical *vs*. polar, ocean *vs*. land, *etc.*). Therefore, the vertical resolution and accuracy of any given retrieval is a function of scene.
- In previous slides we have shown that portions of the retrieval error (e.g. those due to the first guess/trapezoidal smoothing) are beyond the physical retrieval capability.
- It makes sense to use an estimate of the information content on a case-by-case basis for comparisons of retrievals to correlative measurements.
  - Use the averaging kernel/trapezoids to convolve the correlative measurement such the this profile is more comparable to what the retrieval would “see” given that profile.
- WOUDC Ozone/Radiosondes (see M. Divarkarla’s talk 9:10 today)
  - Weighted toward polar cases
  - Water from matched operational radiosonde
- Comparisons are for temperature and water only.



WOUDC Sondes Temperature Statistics
100
1000
Pressure, [mb]
SDV(Ret)
Bias(Ret)
SDV(Cnv Ret)
Bias(Cnv Ret)
−2
−1
0
1
2
3
4
5
Bias / SDV, [K]


WOUDC Sondes Water Statistics
Pressure, [mb]
100
1000
SDV(Ret)
Bias(Ret)
SDV(Cnv Ret)
Bias(Cnv Ret)
-20
0
20
40
60
80
100
Bias / SDV, [%]

Summary

- AIRS averaging kernels and smoothing operators enable "fair" comparison of the physical retrieval to correlative measurements
  - smoothing due to trapezoids
  - smoothing due to damping (averaging kernel)
- Accounting for errors due to trapezoidal smoothing gives a lower limit to retrieval ability.
  - MAX 0.5 K for T
  - MAX 10% for H_2O and O_3
- Averaging kernel derived resolution is similar in vertical shape to resolution derived from error covariance matrices.
  - averaging kernels for the physical temperature and moisture retrievals are good representations of retrieval vertical weighting

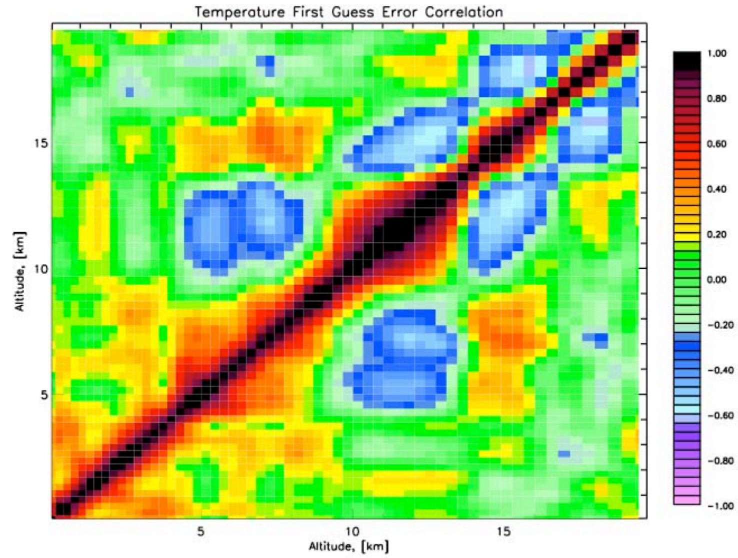
Future Directions

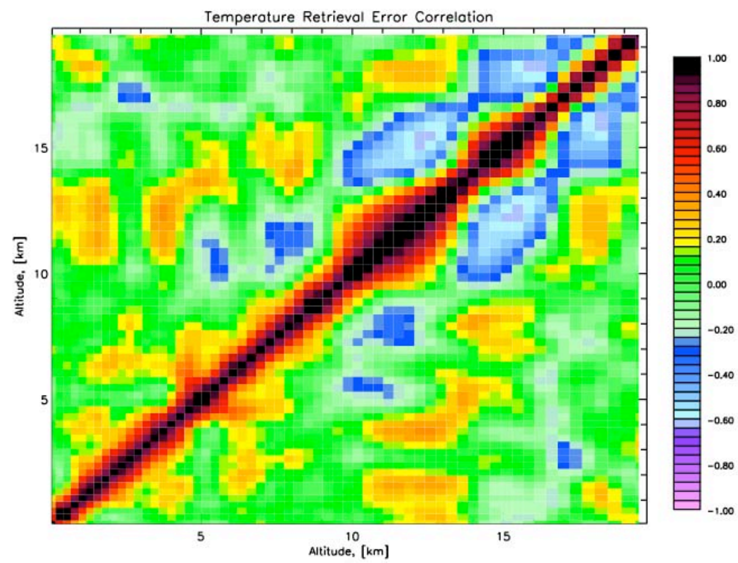
- Publish results (draft in progress).
- Transformation between 100 layer and trapezoidal functions introduces large scale vertical correlation in 100 layer products
  - Consider using more or different retrieval basis functions (*e.g.* triangles vs. trapezoids).
- Analysis of information content for ozone in different scenes.

Questions?

Error Correlation at TWP

- Physical retrieval error correlation (bottom panel) is more diagonal than the regression error correlation (lower panel)
- Smoothing at the tropopause ~15km is evident in both physical and regression solutions.





Derivation of averaging kernels

$$\ddot{\mathbf{O}}_{j,j'} = \mathbf{G}_{j,n} \cdot \mathbf{K}_{n,j'}$$

$$\ddot{\mathbf{O}}_{j,j'} = \mathbf{U}_{j,k} \cdot \mathrm{diag}\left(\frac{\ddot{\mathbf{o}}_k}{\ddot{\mathbf{e}}_k}\right) \cdot \mathbf{U}^T_{k,j'} \cdot \mathbf{K}^T_{j',n} \cdot \mathbf{W}_{n,n'} \cdot \mathbf{K}_{n',j'}$$

$$\ddot{\mathbf{O}}_{j,j'} = \mathbf{U}_{j,k} \cdot \mathrm{diag}\left(\frac{\ddot{\mathbf{o}}_k}{\ddot{\mathbf{e}}_k}\right) \cdot \mathbf{U}^T_{k,j'} \cdot \mathbf{U}_{j,k} \cdot \mathrm{diag}(\ddot{\mathbf{e}}_k) \cdot \mathbf{U}^T_{k,j'}$$

$$\ddot{\mathbf{O}}_{j,j'} = \mathbf{U}_{j,k} \cdot \mathrm{diag}(\ddot{\mathbf{o}}_k) \cdot \mathbf{U}^T_{k,j'}$$